Modeling of coupled core inductors for application in state average buck converter simulations

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Abstract

State average models have reduced significantly the amount of simulation time required in the analysis of switch mode power supplies. Coupled core inductors have improved dynamic cross regulation and reduced the size of the output filter capacitors. The combination of state average models and coupled core inductors in multiple output buck converter simulations has proven to be somewhat inharmonious. This paper will attempt to address the limitations and methods of modeling coupled core inductors in state average simulations.

Introduction:

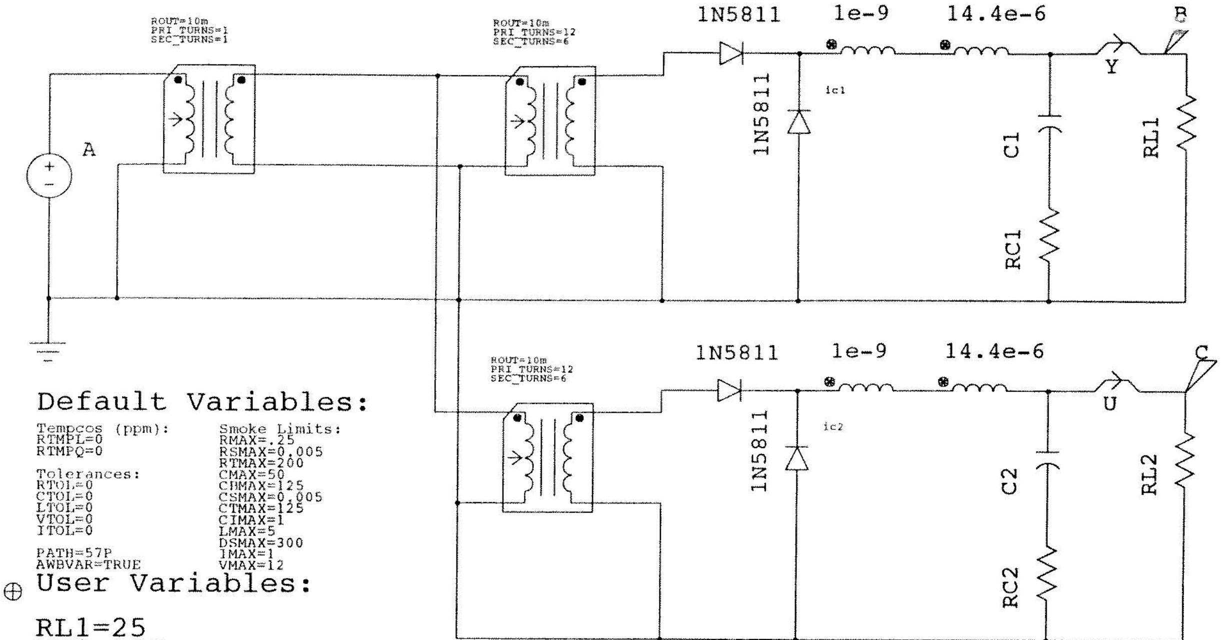
Analog Workbench maintains the capability to simulated switching power supplies. These simulations can be subdivided into two major modeling categories as state average models and detailed switching models. The detailed switching models simulates cycle by cycle response and are inherently slow but accurate. The state average models approximates the switching action as an average. This produces a models which is faster but less accurate.

Many multi-output power supplies being design today rely on coupled core inductors. This paper will focus on the limitations of state average models in modeling coupled core inductors and suggest possible compromises in the simulation of them in state average simulations.

Cross Regulation:

The first question to be asked is 'why are coupled core inductors used in switching power supplies'? One reason is because they improve cross regulation. Cross regulation in multi-output power supplies is the ability of the power supply to compensate for load changes on one output by coupling this change into another output.

To illustrate cross regulation an example dual power output stage will be evaluated with and without coupling. Figure #1 shows the results of independent uncoupled outputs. (An initial condition on both inductors of 200 ma and 50 ma respectively is used to demonstrate load differences.) As can be seen in Figure #1 the "C" and "B" outputs do not track but actually appear to diverge.



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Tolerances:
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ITOL=0

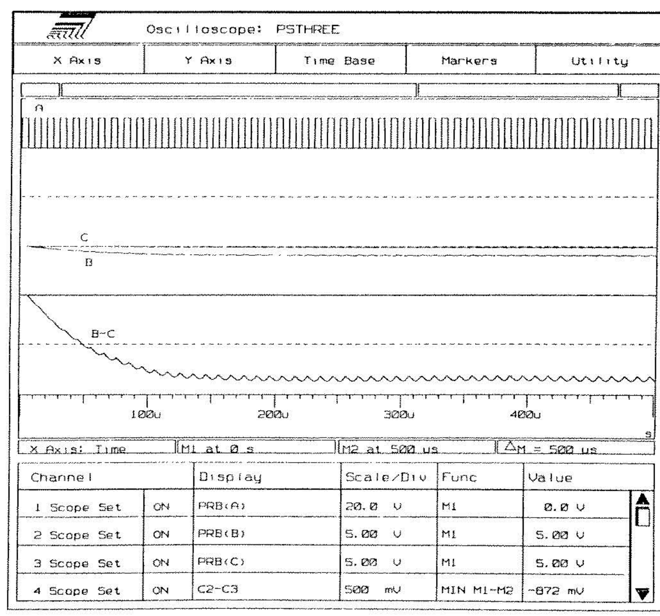
PATH=57P
AWBVAR=TRUE

Smoke Limits:
RMAX=.25
RSMAX=0.005
RTMAX=200
CMAX=50
CBMAX=125
CSMAX=0.005
CTMAX=125
CIMAX=1
LMAX=5
DSMAX=300
IMAX=1
VMAX=12

User Variables:

RL1=25
RL2=100
IC1=5.0/RL1
IC2=5.0/RL2
AL=200e-3
CP=0.99
C1=10e-6
RC1=0.02
C2=C1
RC2=RC1

Figure #1
Transient Response of a
Dual Power Output Stage
With Independent Inductors



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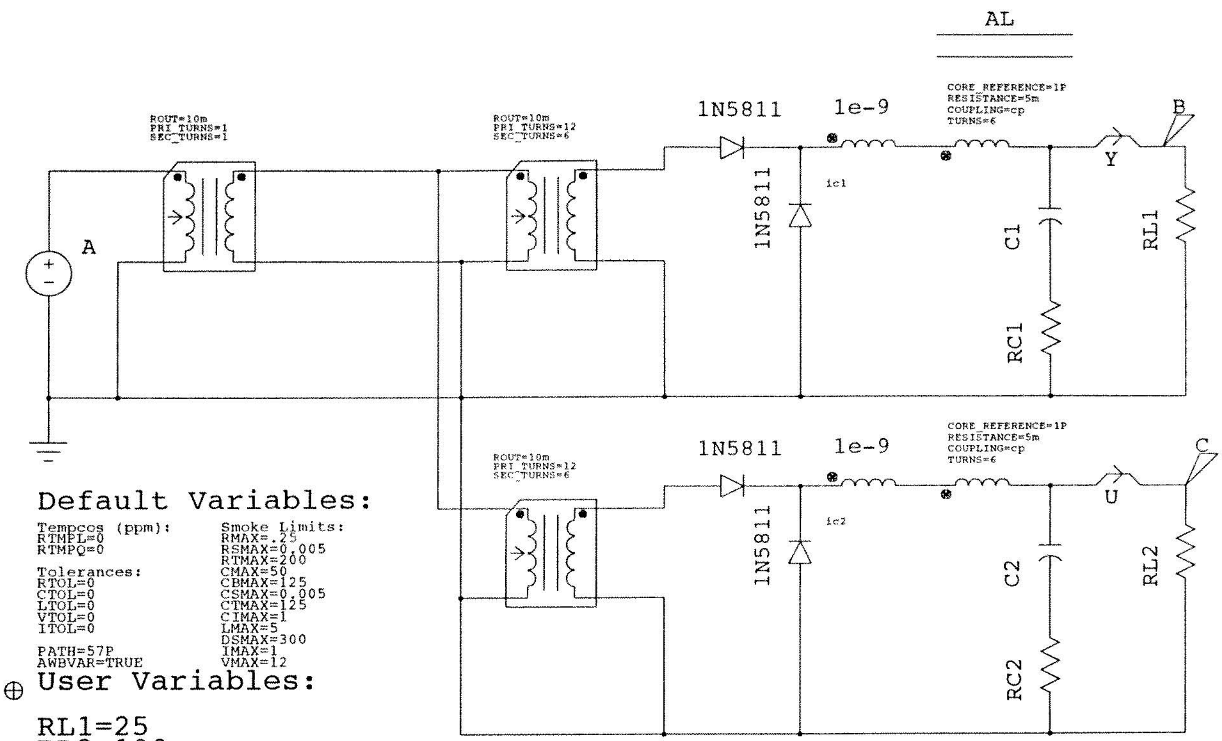
2 Scope Set	ON	PRB(B)	5.00 V	M1	5.00 V
3 Scope Set	ON	PRB(C)	5.00 V	M1	5.00 V
4 Scope Set	ON	C2-C3	500 mV	MIN M1-M2	-872 mV

In Figure #2 the same initial conditions are used with an equivalent coupled core inductor used in place of the independent inductors. The results suggest that there is a significant improvement in cross regulation because the two outputs, "C" and "B", although loaded differently have approximately the same steady state dc output voltage level.



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RTMPQ=0

Tolerances:
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CTOL=0
LTOL=0
VTOL=0
ITOL=0

PATH=57P
AWBVAR=TRUE

Smoke Limits:
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RSMAX=0.005
RTMAX=200
CMAX=50
CBMAX=125
CSMAX=0.005
CTMAX=125
CIMAX=1
LMAX=5
DSMAX=300
IMAX=1
VMAX=12

User Variables:

RL1=25
RL2=100
IC1=5.0/RL1
IC2=5.0/RL2
AL=200e-3
CP=0.99
C1=10e-6
RC1=0.02
C2=C1
RC2=RC1

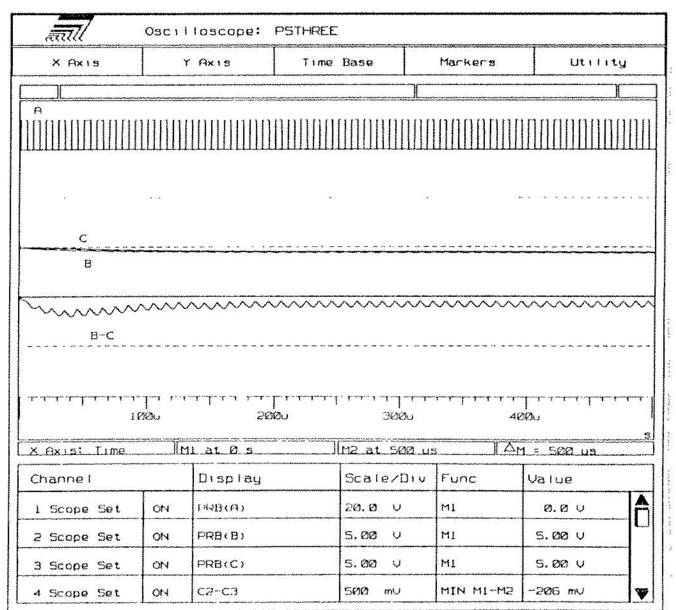


Figure #2
Transient Response of a
Dual Power Output Stage
With a Coupled Core Inductor

Reflected Inductance:

Another reason coupled core inductors are used is because the they reduce the required size of the output filter capacitors needed to filter the ripple currents of the various outputs. Typically, the inductance needed to individually filter each output is larger than the common inductance of the coupled core inductor. This can be evaluated by comparing the reflected inductance of the coupled core inductor to the independent discrete inductors.

The reflected inductance for a multi-output power stage with a common core inductor can be determined by using the following formula

$$L_{reflected} = AL*Ni^2\left[\frac{Np^2}{Ns^2}\right] \quad (1)$$

Where Np is the number of turns on the primary of the power transformer, Ns is the number of secondary turns on the transformer and Ni is the number of turns on the coupled core inductor. (In most applications Ni is equal to Ns.)

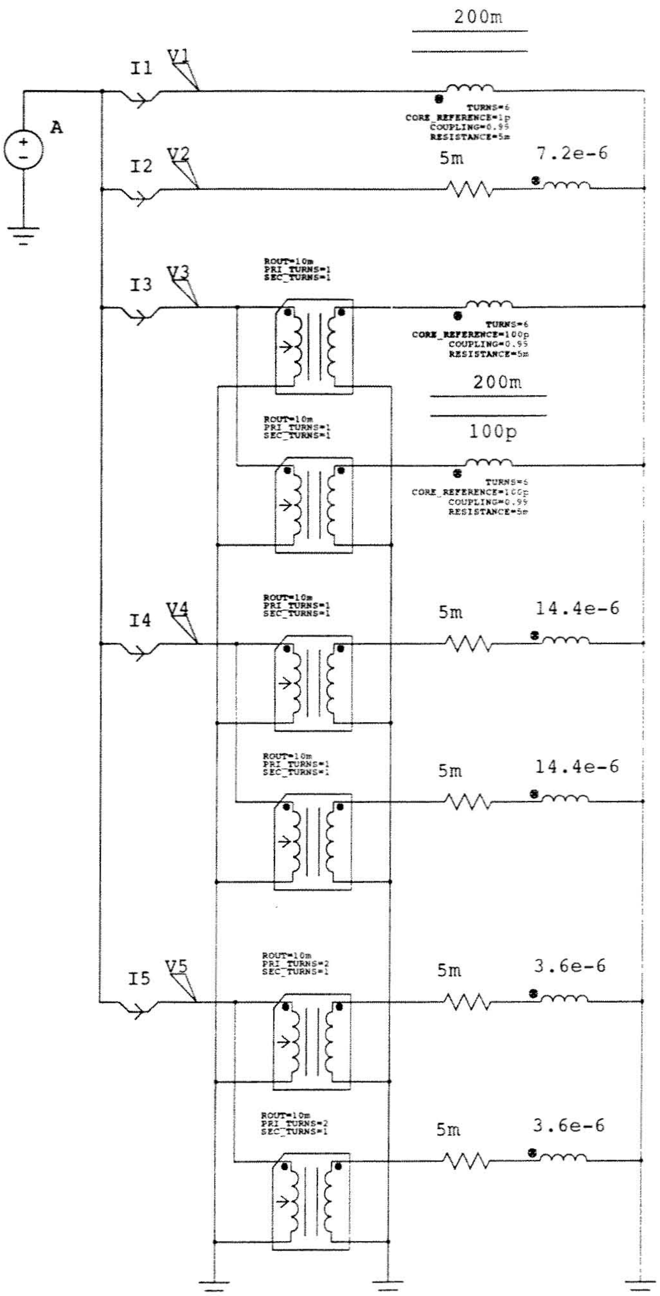
AL is the inductance factor. This is usually expressed in so many milliheneries per thousand turns. The designer should understand that inductance is related to turns squared.

The reflected inductance for a dual power output stage with discrete inductors can be determined by using the following formula

$$L_{reflected} = L1\left[\frac{Np^2}{Ns1^2}\right] \,\Big|\Big|\, L2\left[\frac{Np^2}{Ns2^2}\right] \quad (2)$$

Where Np is the number of turns on the primary of the power transformer, Ns1 is the number of secondary turns on the transformer for the L1 output and Ns2 is the number of secondary turns on the transformer for the L2 output.

A comparison of equations 1 and 2 shows that the discrete inductors must have twice the inductance of the coupled core inductor to have the same reflected inductance for a dual power output stage.



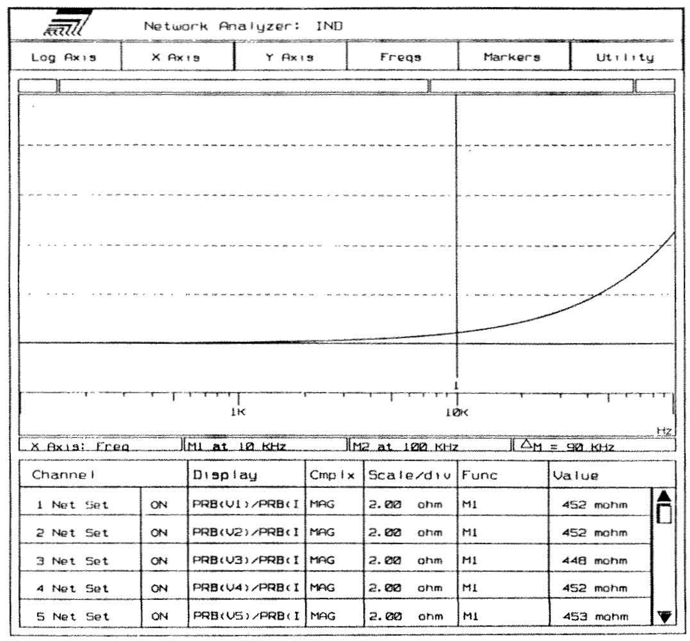


Figure #3
Comparison Between Equivalent Reflected Inductance Circuits

The use of state average models to simulate multi-output buck topology switching power supplies requires the reflection of output inductance to the primary of the power transformer.

To further investigate reflected inductance the following example will be used. A frequency sweep of five equivalent circuits with the same reflected inductance is shown in Figure #3. First, it can be seen in Figure #3 that

$$V1/I1 = V2/I2 \qquad (3)$$

or

$$7.2uH = \frac{200mH*(6\ turns)^2}{(1000\ turns)^2} \qquad (4)$$

Therefore, the statement made earlier regarding "inductance being related to turns squared" is validated. Second, its can be seen that multiple outputs which have a coupled core inductor in their outputs would have the same reflected inductance as defined in equation #3 or

$$V3/I3 = V1/I1 \qquad (5)$$

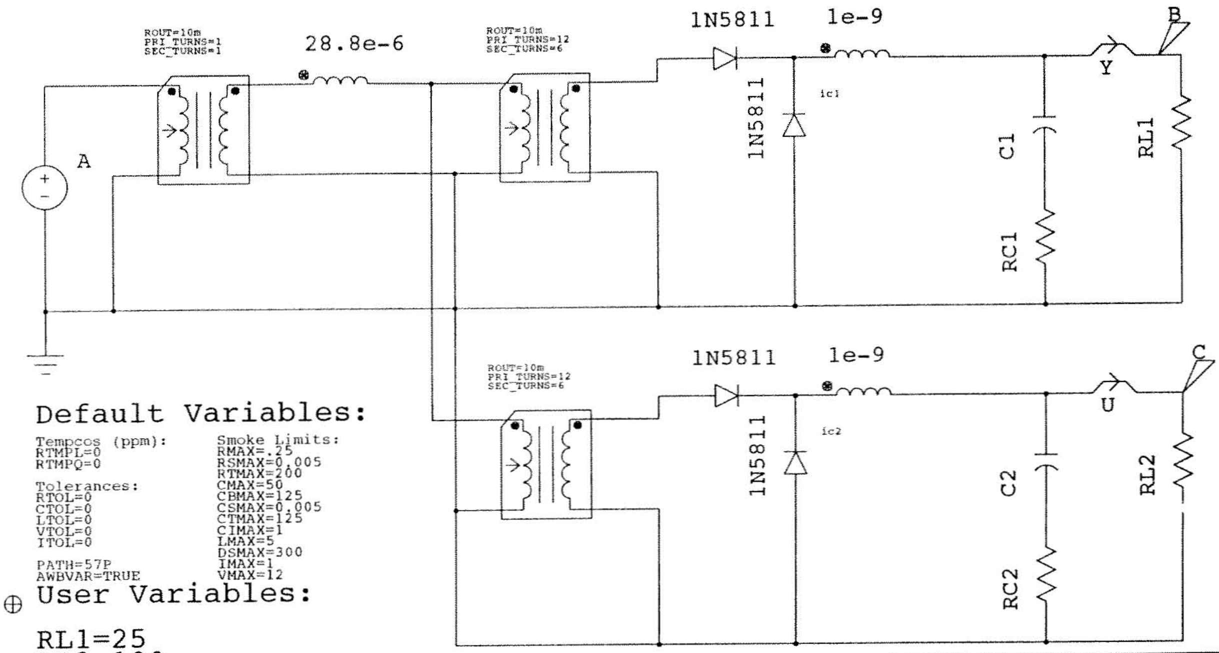
Third, independent output inductors must be placed in parallel and then reflected to the primary of the transformer. Thus, individual inductors in Figure #3 have twice the inductance of the coupled core inductors or

$$V4/I4 = V3/I3 \qquad (6)$$

as was defined in detail in equation 2. Forth, the reflected inductance is related to the turns ratio squared of the primary to secondary windings of the transformer or

$$V5/I5 = L1\left[\frac{Np^2}{Ns1^2}\right] \Big|\Big| \; L2\left[\frac{Np^2}{Ns2^2}\right] \qquad (7)$$

Thus, it can be seen that reflected inductance in each of the equivalent circuits is equal. However, as was shown in Figures #1 and #2 the transient responses of coupled and uncoupled circuits would have the same reflected inductance but produce very different transient results. Therefore, the reflected inductance model in its present form would not be able to simulate the uncoupled reflected condition but it would be able to simulate the coupled condition. Figure #4 shows that the equivalent reflected inductance output stage of Figure #1 and #2. The transient response of Figure #4 corresponds to the transient response of Figure #2 but not Figure #3.



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RTMPQ=0

Tolerances:
RTOL=0
CTOL=0
LTOL=0
VTOL=0
ITOL=0

PATH=57P
AWBVAR=TRUE

Smoke Limits:
RMAX=.25
RSMAX=0.005
RTMAX=200
CMAX=50
CBMAX=125
CSMAX=0.005
CTMAX=125
CIMAX=1
LMAX=5
DSMAX=300
IMAX=1
VMAX=12

User Variables:

RL1=25
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IC2=5.0/RL2
AL=200e-3
CP=0.99
C1=10e-6
RC1=0.02
C2=C1
RC2=RC1

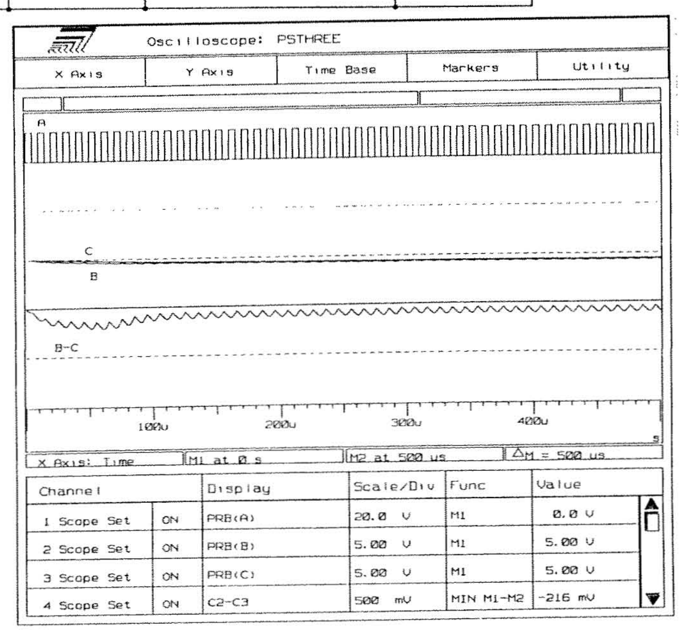


Figure #4
Transient Response of a
Dual Power Output Stage
With Inductance Reflected

State Average Comparison

A comparison between the state average model and switching model of the dual power output stage shown in Figure #2 is now needed to complete the picture. Figure #5 shows the transient response of the equivalent state average model. (The reflected inductance is 28.8 uH, the duty cycle is 50% and the input voltage is 12 VDC.) The reflected inductance is absorbed into the PWMBCKD DC to DC transformer as a part parameter. As can be seen the response shown in Figure #5 is very similar to the response shown in Figure #2 but it does not correspond to Figure #1.

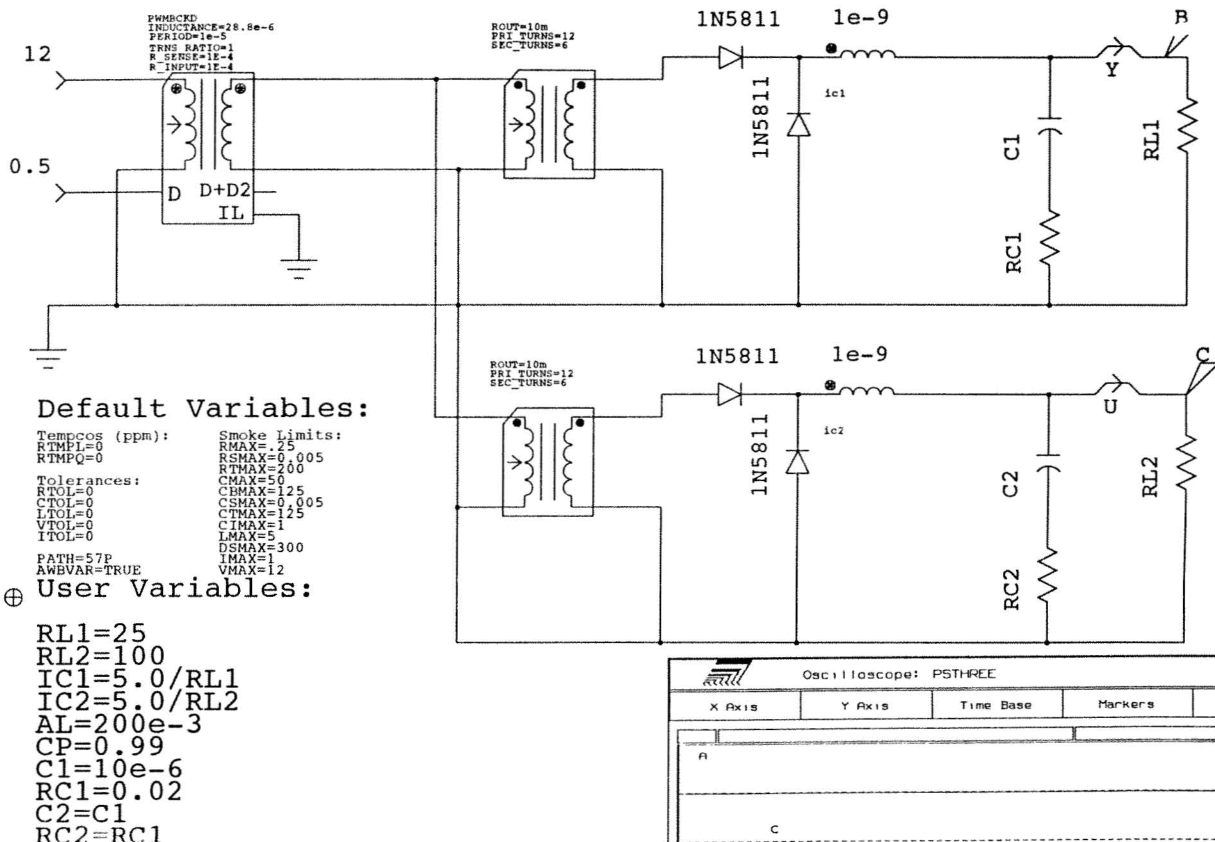
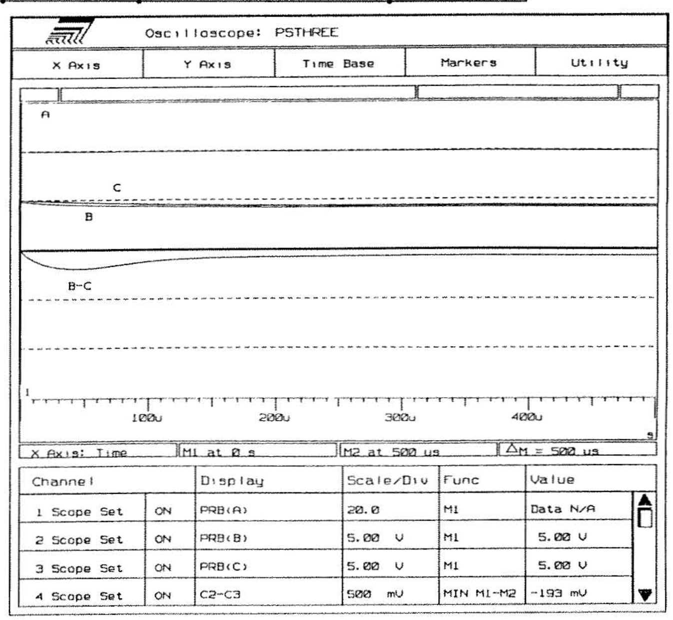
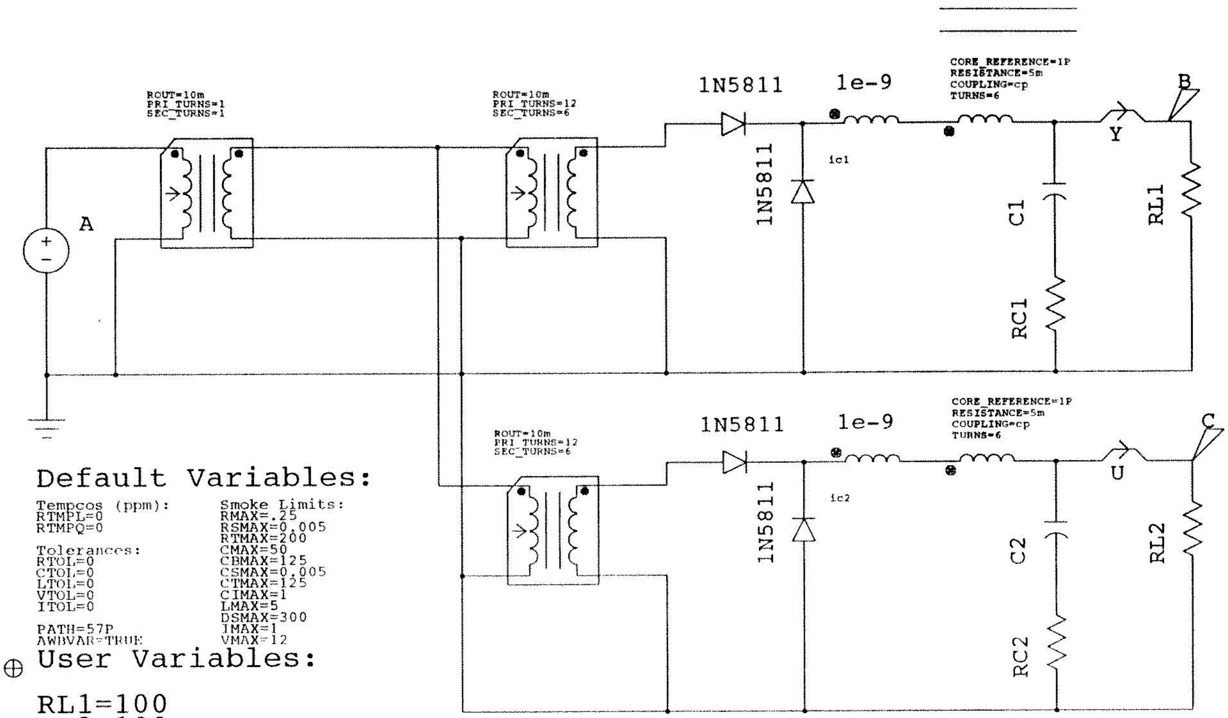


Figure #5
State Average Response of a
Dual Power Output Stage



Frequency Response:

Next, consider how the small signal AC frequency response of the coupled core inductor in the dual power output stage differs from its equivalent reflected inductance model. Figure #6 shows the open loop frequency response of two identical outputs which are loaded the same and have a coupled core inductor in their outputs. The frequency responses of B/A and C/A are both 3dB @ 18.1KHz and -166 degrees @ M2.



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Tolerances:
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CTOL=0
LTOL=0
VTOL=0
ITOL=0

PATH=57P
AWBVAR=TRUE

Smoke Limits:
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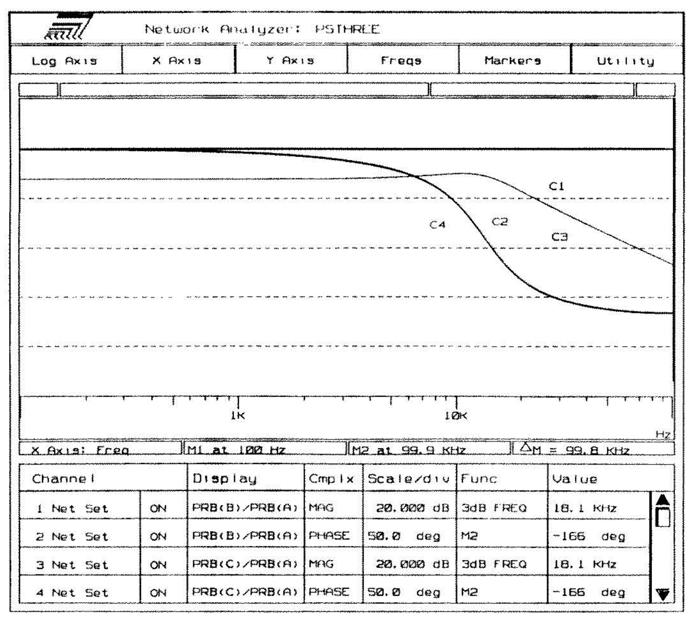
RSMAX=0.005
RTMAX=200
CMAX=50
CBMAX=125
CSMAX=0.005
CTMAX=125
CIMAX=1
LMAX=5
DSMAX=300
IMAX=1
VMAX=12

User Variables:

RL1=100
RL2=100
IC1=5.0/RL1
IC2=5.0/RL2
AL=200e-3
CP=0.99
C1=10e-6
RC1=0.02
C2=C1
RC2=RC1



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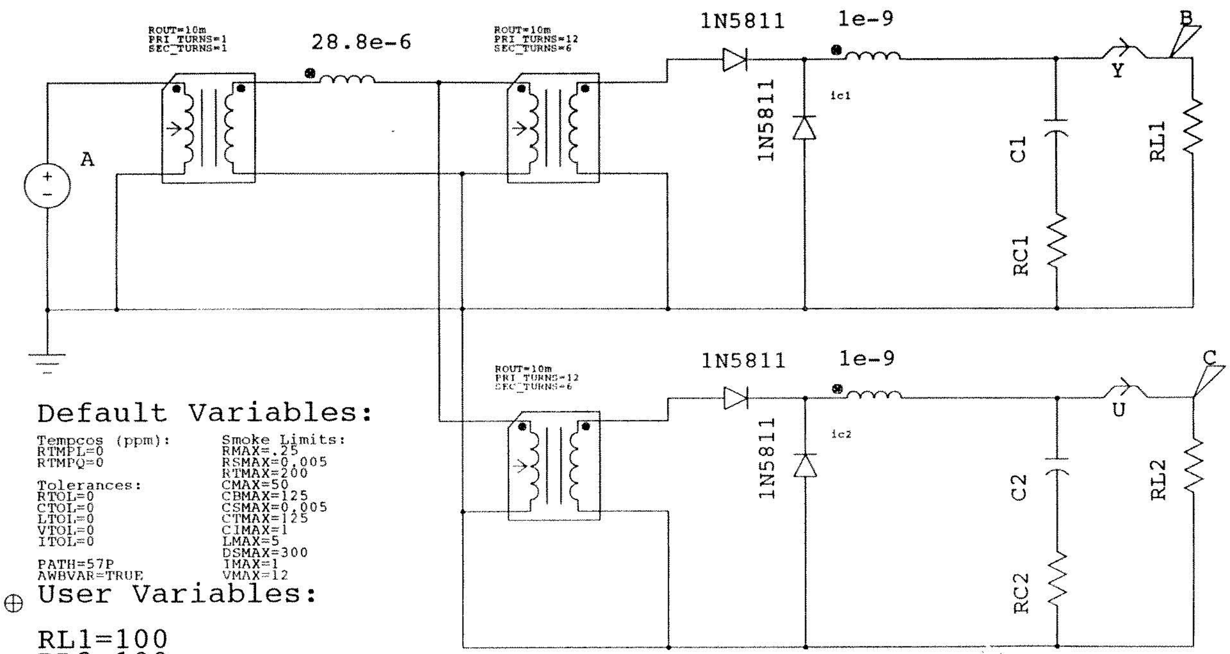
2 Net Set	ON	PRB(B)/PRB(A)	PHASE	50.0 deg	M2	-166 deg
3 Net Set	ON	PRB(C)/PRB(A)	MAG	20.000 dB	3dB FREQ	18.1 KHz
4 Net Set	ON	PRB(C)/PRB(A)	PHASE	50.0 deg	M2	-166 deg

Figure #6
Frequency Response of a
Dual Power Output Stage
With a Coupled Core Inductor

Figure #7 shows the frequency response of two identical outputs which are again loaded the same and have a common reflected inductance in the primary of the power transformer. The frequency responses of B/A and C/A are again identical, 3dB @
16.2KHz and -155 degrees @ M2, but they differ from the response shown in Figures #6.



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Tolerances:
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AWBVAR=TRUE

Smoke Limits:
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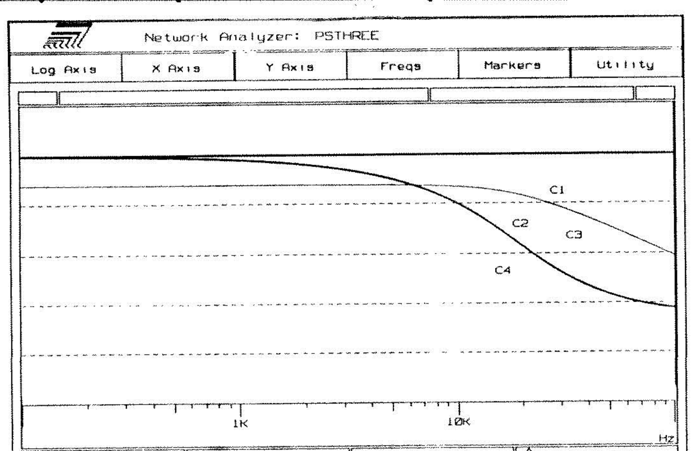
RSMAX=0.005
RTMAX=200
CMAX=50
CBMAX=125
CSMAX=0.005
CTMAX=125
CIMAX=1
LMAX=5
DSMAX=300
IMAX=1
VMAX=12

User Variables:

RL1=100
RL2=100
IC1=5.0/RL1
IC2=5.0/RL2
AL=200e-3
CP=0.99
C1=10e-6
RC1=0.02
C2=C1
RC2=RC1

Figure #7
Frequency Response of a
Dual Power Output Stage
With Inductance Reflected



Channel		Display	Cmplx	Scale/div	Func	Value
1 Net Set	ON	PRB(B)/PRB(A)	MAG	20.000 dB	3dB FREQ	16.2 KHz
2 Net Set	ON	PRB(B)/PRB(A)	PHASE	50.0 deg	M2	-156 deg
3 Net Set	ON	PRB(C)/PRB(A)	MAG	20.000 dB	3dB FREQ	16.2 KHz
4 Net Set	ON	PRB(C)/PRB(A)	PHASE	50.0 deg	M2	-156 deg

A comparison between the four topologies (state average, coupled core, uncoupled inductors and reflected inductance) shows that from a small signal stand point the state average model has the same response as the reflected inductance model and the coupled core inductor model has the same response as the uncoupled inductor model. This is shown in Figure #8.

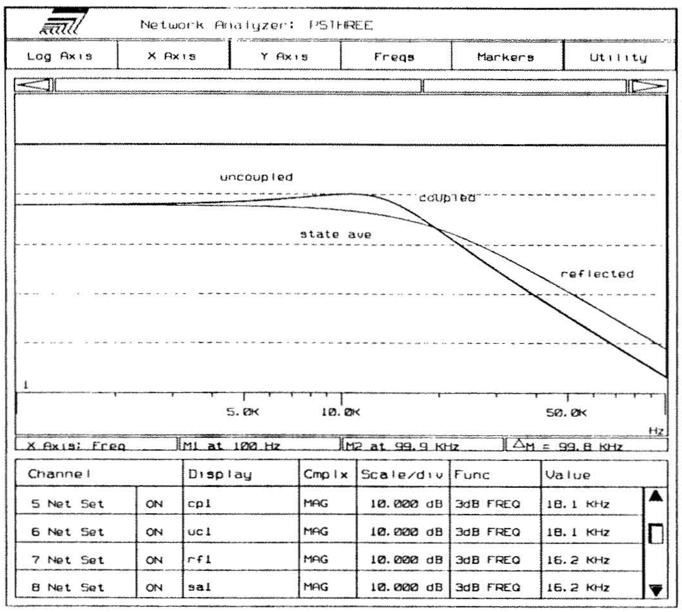
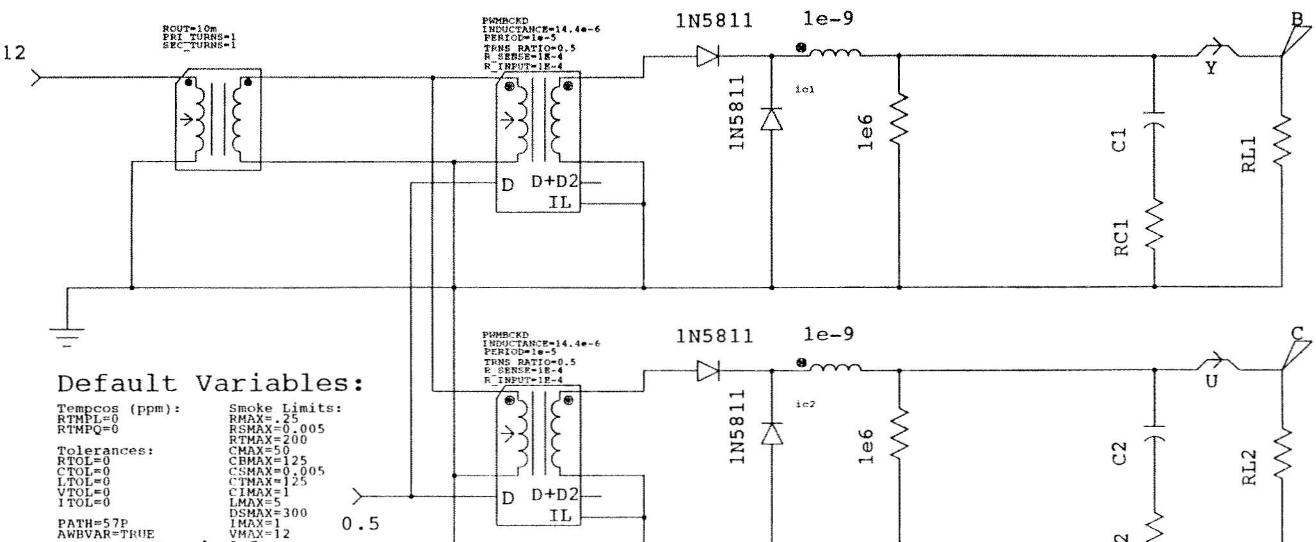


Figure #8
Comparison Between the
Frequency Responses of the
Four Topologies of the
Dual Power Output Stage

Coupled and uncoupled core state average models:

The transient response of the state average model of a dual power output stage is equivalent to the switched response of the coupled core model. However, the ac response of the state average model does not completely model the ac response of the dual power output stage. Therefore, additional models must be developed which will completely model coupled core inductors in both domains.

In Figure #1 the uncoupled or discrete inductor power output stage had outputs which did not track. A proposed state average model model of the uncoupled power output stage includes two PWMBCKD DC to DC transformers. These transformers are connected up in place of the secondaries of the power transformer. The uncoupled inductors are absorbed into the PWMBCKD models. Figure #9 and #10 shows the transient and small signal ac responses of a state average model for the discrete inductor power output stage respectively. The duty cycle and period are equal to those values used in Figure #4 and the turns ratio equals the primary to secondary turns ratio used in the switching model for the secondaries.



User Variables:

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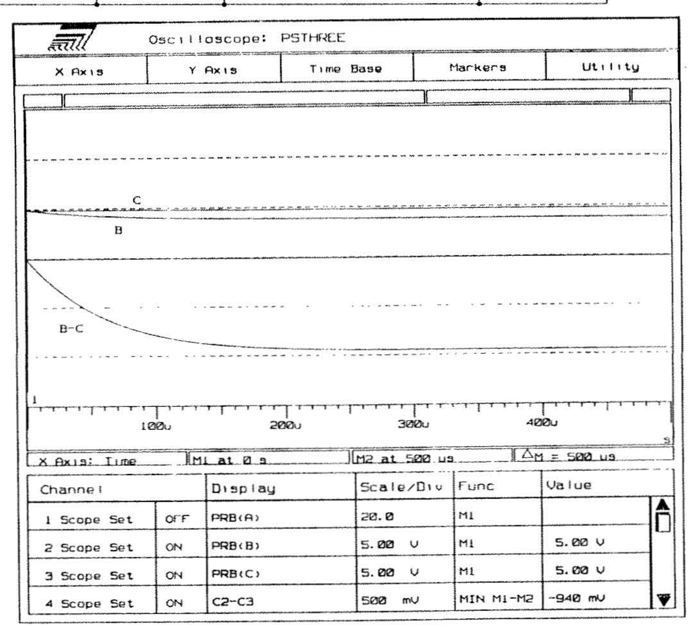
RL2=100
IC1=5.0/RL1
IC2=5.0/RL2
AL=200e-3
CP=0.99
C1=10e-6
RC1=0.02
C2=C1
RC2=RC1

Figure #9
State Average Response of a
Dual Power Output Stage
With Independent Inductors



By using the PWMBCKD model the designer can also determine quite easily whether the individual inductor currents are continuous. The state average transient response shown in Figure #9 compares closely to the switching transient response shown in Figure #1. However, there still seems to be some high frequency error in the small signal response when comparing Figures #10 to #6. This may in part be due to the discontinuous inductor current.

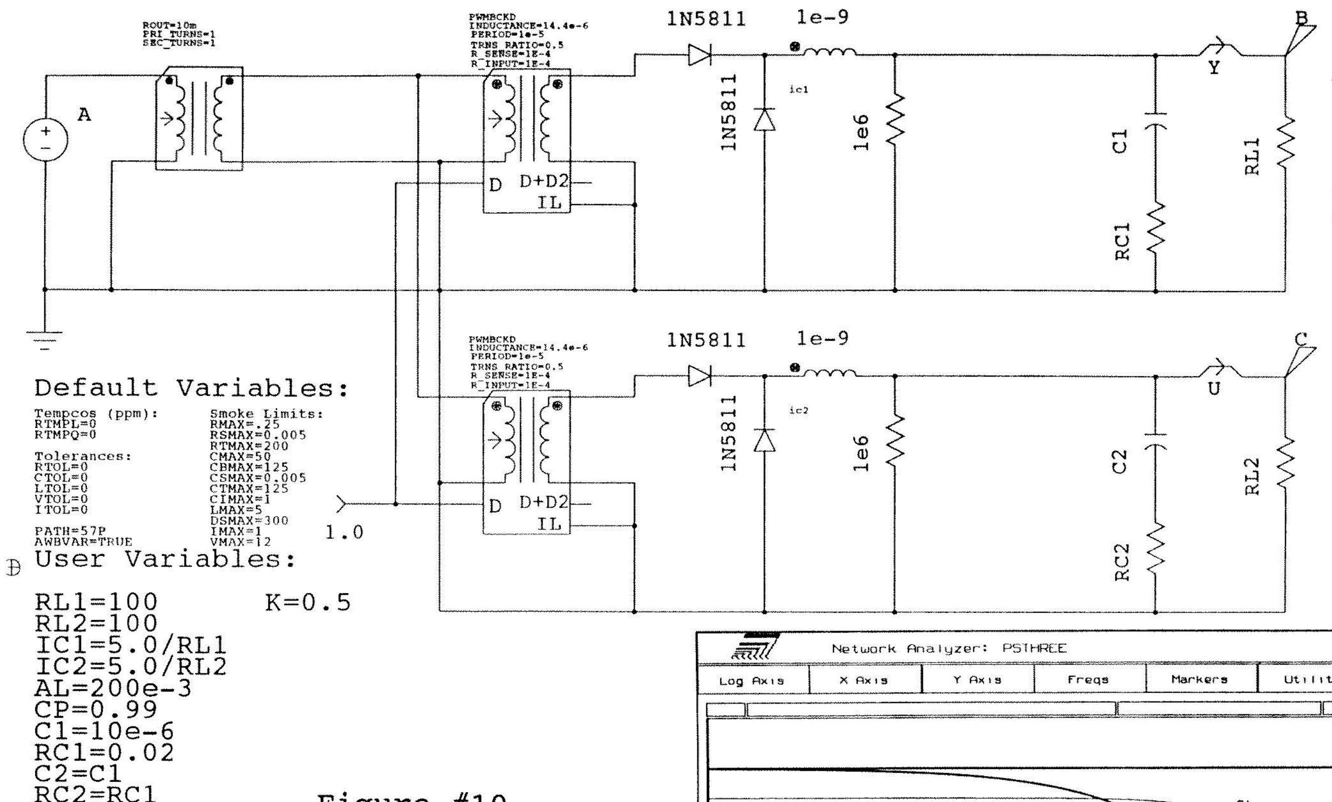
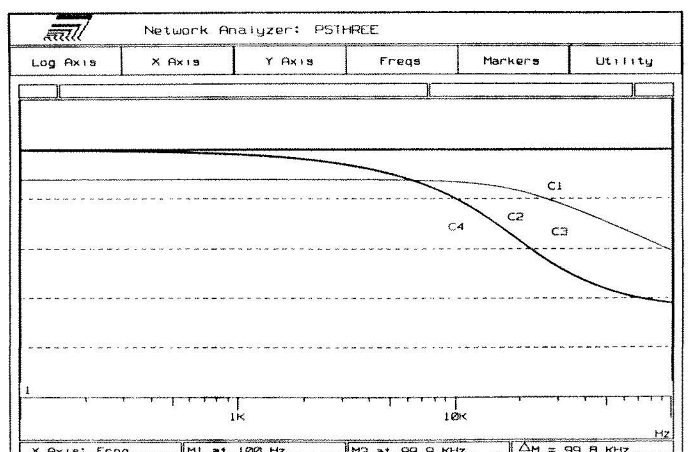


Figure #10
Frequency Response of a
Dual Power Output Stage
With Independent Inductors
Using the State Average Model



Channel		Display	Cmplx	Scale/div	Func	Value
1 Net Set	ON	PRB(B)/PRB(A)	MAG	20.000 dB	3dB FREQ	16.2 KHz
2 Net Set	ON	PRB(B)/PRB(A)	PHASE	50.0 deg	M2	-155 deg
3 Net Set	ON	PRB(C)/PRB(A)	MAG	20.000 dB	3dB FREQ	16.2 KHz
4 Net Set	ON	PRB(C)/PRB(A)	PHASE	50.0 deg	M2	-155 deg

A first order model of a coupled core inductor was developed which would attempt to mimic the transient response of Figure #5 but would have the same small signal response as shown in Figure #6. Again the dual PWMBCKD DC to DC transformers were used to model the state average performance but an additional element was added to account for the cross coupling. A model was developed which compared the difference in output voltages and added a shunt current based on the difference between the outputs. A first order lag was included in the voltage controlled current sources' path to account for the 3dB point of the power output stages. The state average model is shown in Figure #11. The inductance used in the PWMBCKD was 14.4uH. This model was then integrated into the

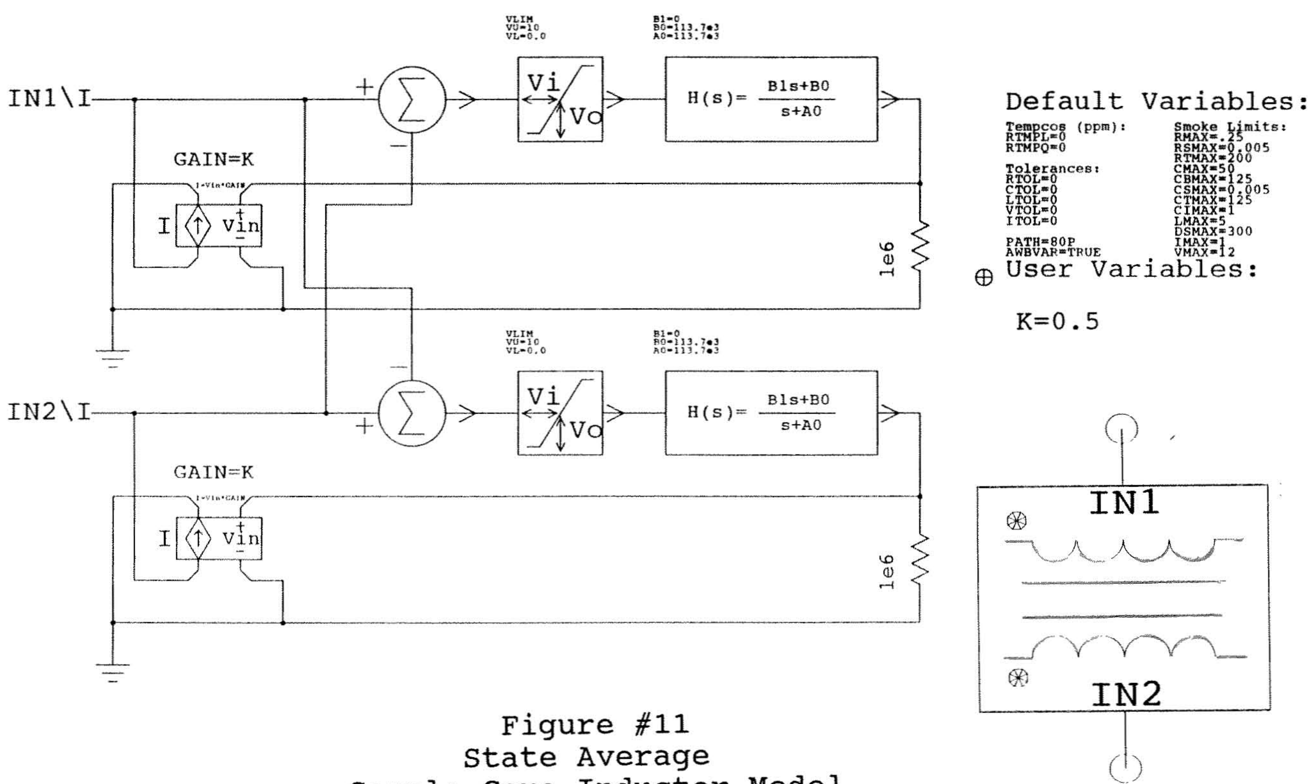


Figure #11
State Average
Couple Core Inductor Model

uncoupled state average model and the results are shown in Figures #9 and #10. Again, the state average transient response shown in Figure #11 compares closely to the switching transient response shown in Figure #5. However,

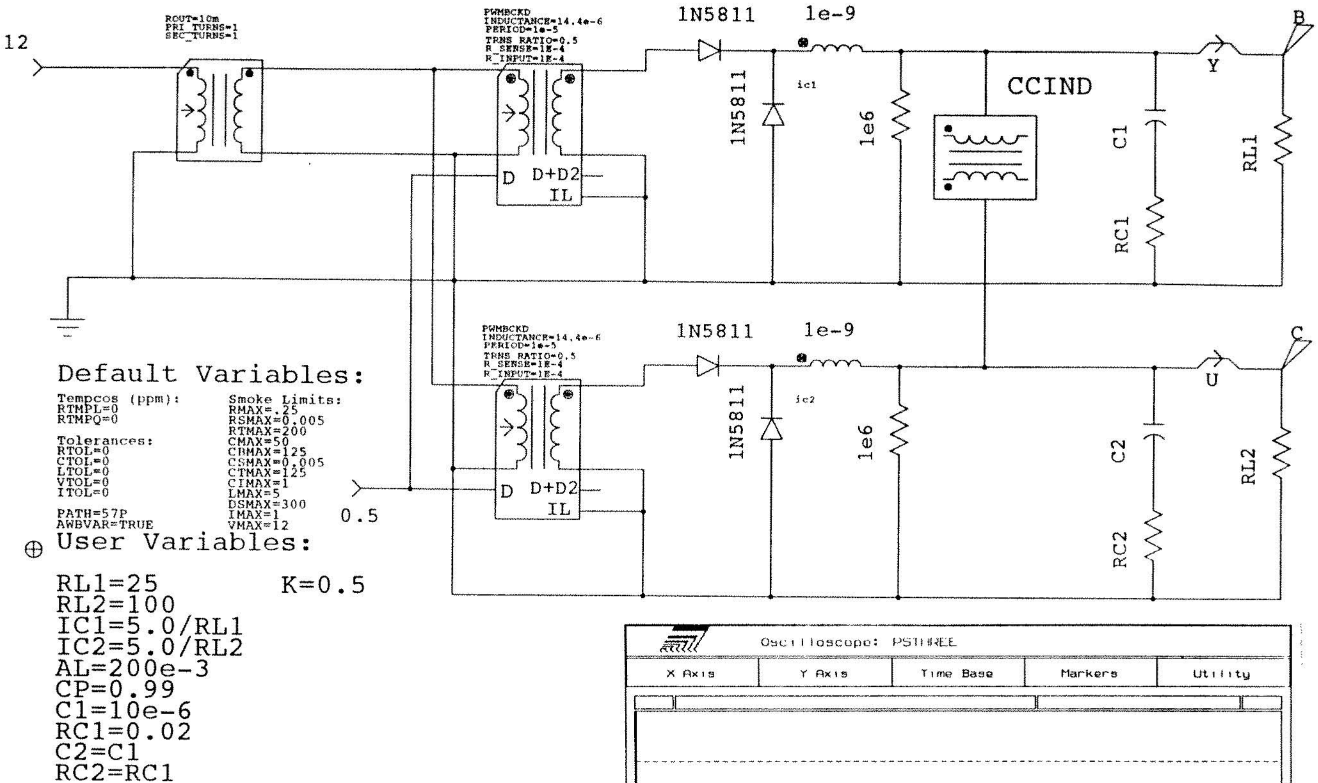
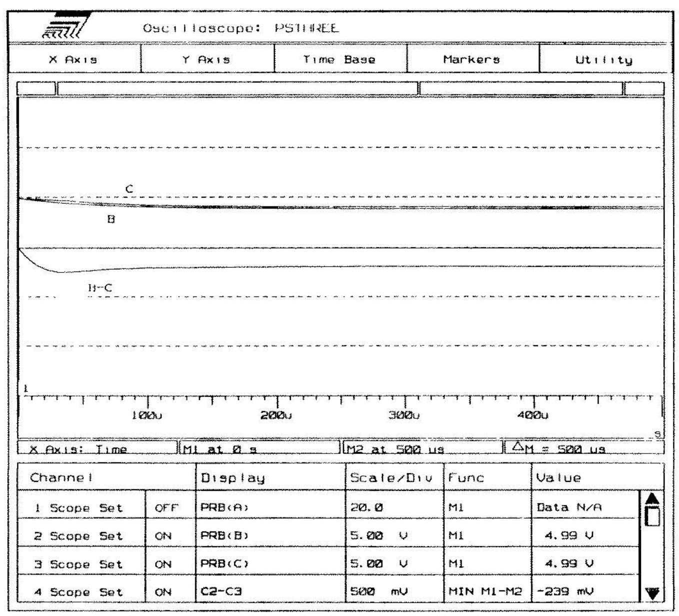
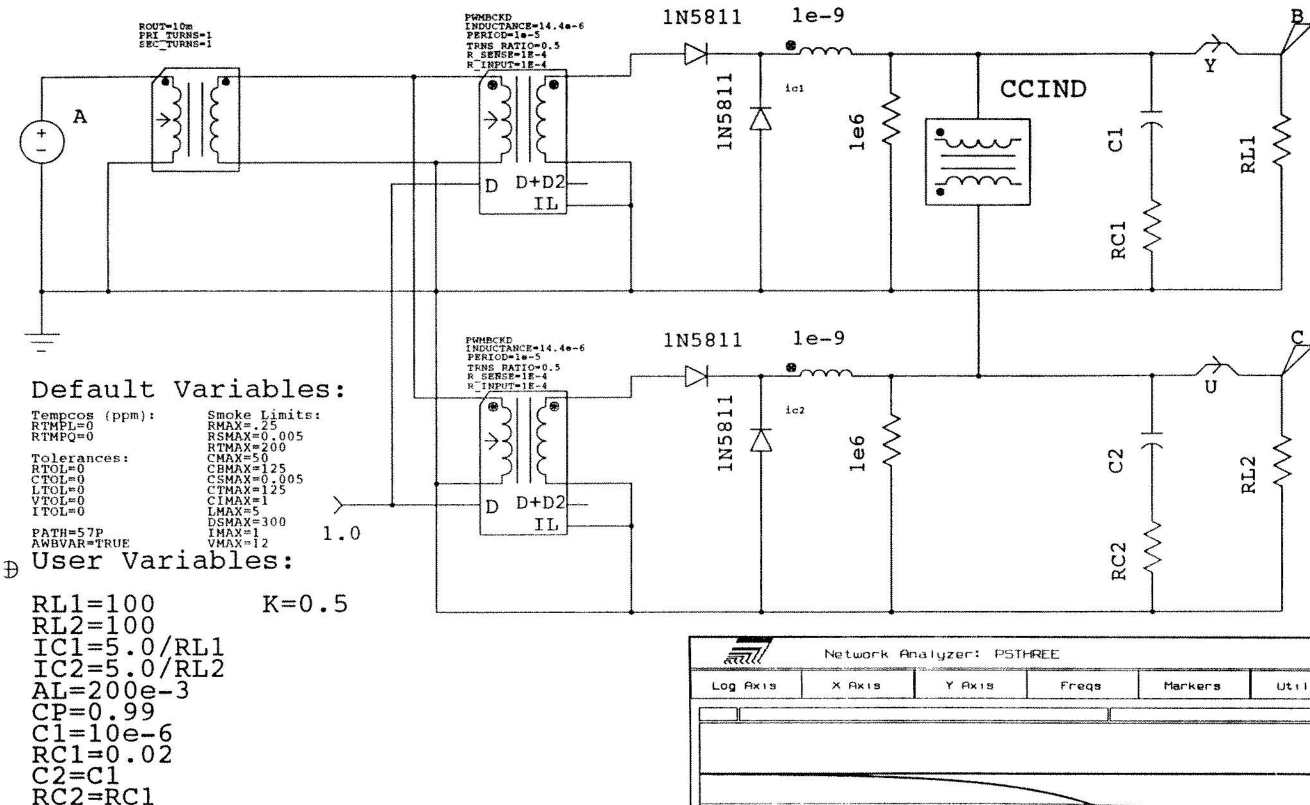
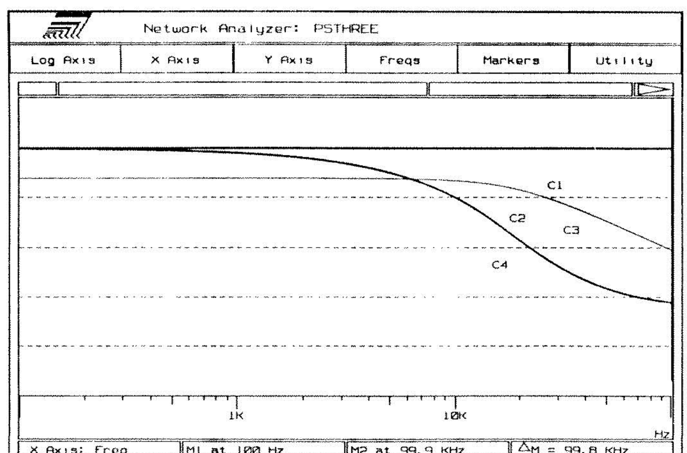


Figure #12
State Average Response of a
Dual Power Output Stage
With a State Average
Couple Core Inductor Model



there still seems to be some high frequency error in the small signal response when comparing Figures #13 to #6. This may in part be due to the discontinuous inductor current as was present in the uncoupled state average model.





Channel		Display	Cmplx	Scale/div	Func	Value
1 Net Set	ON	PRB(B)/PRB(A)	MAG	20.000 dB	3dB FREQ	16.2 KHz
2 Net Set	ON	PRB(B)/PRB(A)	PHASE	50.0 deg	M2	-156 deg
3 Net Set	ON	PRB(C)/PRB(A)	MAG	20.000 dB	3dB FREQ	16.2 KHz
4 Net Set	ON	PRB(C)/PRB(A)	PHASE	50.0 deg	M2	-156 deg

Figure #13
Frequency Response of a
Dual Power Output Stage
With a State Average
Couple Core Inductor Model

Conclusions:

It appears that the state average models developed and used in Figure #9 and #11 could be used in transient simulations. However, they lack the completeness needed to account for the small signal response.

The modeling of coupled core inductors in state average models has proven to be somewhat difficult. However, the use of these type of inductors in switching power supplies will persist and models must be developed which will represent the state average equivalent circuits.

References:

1.) "Designing DC/DC Converters with the Si9110 Switchmode Controller", AN88-3, Siliconix Incorporated, August 1988 by James Blanc

2.) "Topics in Multiple-Loop Regulators and Current-Mode Programming", IEEE Power Electronics Specialists Conference, 985 Record, pp 716-732, by R. D. Middlebrook

3.) "Coupled Filter Inductors in Multiple Output Buck Regulators Provide Dramatic Performance Improvements", Unitrode Power Supply Design Seminar, SEM-800, M7